Menoufia University Faculty of Engineering Shebin El-Kom Dep. of Basic Engineering Sciences First Semester Examination	جامعة المنوفية	Subject: Math. (1-A) Code: BES 011 Time Allowed: 3 hours Total Marks: 100 marks Date of Exam: 31/12/2017

(Question Number-1) :(50 Marks)

1- if $A=\begin{bmatrix}1 & 3\\ 2 & -4\end{bmatrix}$

a) Verify Clayey-Hamilton theorem for the matrix A.

b) Diagonalize A; if it is possible.

c) Find A^n and A^{10}.

d) Show whether the following vectors are dependent or independent;

$$\mathbf{u}_1=(1,1,2,1),\ \mathbf{u}_2=(0,2,1,1),\ \mathbf{u}_3=(3,1,2,0)$$

2- a) Use the binomial theorem to find the approximation of $\dfrac{\sqrt{1+x}}{\sqrt{1+3x}}$, neglecting x^3.

b) Find the expansion of $f(x)=\dfrac{27x^2+32x+16}{(3x+2)^2(1-x)}$, in ascending power of x, up to and including the term in x^3, also find the condition of expansion.

3- Use the mathematical induction to prove:

a) n^3+2n is divisible by 3.

b) $\dfrac{5}{1.2.3}+\dfrac{6}{2.3.4}+\dots\dots+\dfrac{n+4}{n(n+1)(n+2)}=\dfrac{n(3n+7)}{2(n+1)(n+2)}$

4- a) A polynomial $P(x)$ has the following roots: $2, 1+\sqrt{3}, 5i$, what is the smallest degree that $P(x)$ could have? Find $P(x)$.

b) If $f(x)=x^4-3x^3-ax^2+bx-52=0$, has $(3+2i)$ as a root, and a, b are real values. Find the other roots and the values of a, b.

c) Resolve into partial fractions $\dfrac{x^4}{x^3+1}$.

(Question Number-2):(50 Marks)

1- Find y' of the following function in simplest form:

1) $y=x^{(\sin^{-1}x)^{\cos x}}$	2) $y=\sec(\ln 3x)+\cosh^{-1}(\log_4 2x)+(\tanh x^2)^{-1}$
3) $y=\ln\left(\sinh^{-1}(3x+1)\right)$	4) $y=\tan^4\left(2+\dfrac{(\cot x)\sqrt{3x+5}}{x^3+\sin^{-1}x}\right)$

2- Find y' of the implicit function: $x^y y^x=(x+y)^{x+y}$ by using two different methods.

3- If $y=\cosh^{-1}\left(\dfrac{1+x}{1-x}\right)$, prove that $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{x}(1-x)}$.

4- If $y=a\cos(\ln x)+b\sin(\ln x)$ prove that:

$$x^2y_{n+2}+x(2n+1)y_{n+1}+(n^2+1)y_n=0.$$

5- Find the n$^{\text{th}}$ derivative of the following function $y=e^{3x}\cos^2 x\sin x$.

6- Prove that $\coth^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right)$, $|x|>1$.

7- If $z=x^2\operatorname{sech}y+y^2\sec x$ and $x=\ln(2t\cos\theta)$, $y=e^{3t\sin\theta}$, Find $\dfrac{\partial z}{\partial\theta}$.

8- Find the following limits using L'Hospital's rule:

a) $\lim\limits_{x\to0}\left(\ln(1-\cos x)-\ln x\right)$ b) $\lim\limits_{x\to0^+}(\sin x)\ln x$ c) $\lim\limits_{x\to+\infty}\left(1+\dfrac{e}{x}\right)^{\frac{x}{2}}$

Good luck

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